Color Correction

is a global change in the coloration of an image to alter its tint, its hues, or the saturation of its colors with minimal changes to its luminant features.
Gamma Adjustment of Color Bands

David Peters, producer, and representatives of the IA, The International Alliance of Theatrical Stage Employees, Moving Picture Technicians, Artists and Allied Crafts, on the set of Frozen Impact (PorchLight Entertainment, 2003).
Gamma Adjustment of Color Bands

red $\gamma = 2$
Gamma Adjustment of Color Bands

Dave & IA image (original); RGBK Histograms

Gamma Adjustment of Color Bands

red $\gamma = 0.5$

reduced red = increased cyan
Gamma Adjustment of Color Bands

David Peters, producer, and representatives of the IA, The International Alliance of Theatrical Stage Employees, Moving Picture Technicians, Artists and Allied Crafts, on the set of Frozen Impact (PorchLight Entertainment, 2003).
Gamma Adjustment of Color Bands

\[ \text{green } \gamma = 2 \]
Gamma Adjustment of Color Bands

David Peters, producer, and representatives of the IA, The International Alliance of Theatrical Stage Employees, Moving Picture Technicians, Artists and Allied Crafts, on the set of Frozen Impact (PorchLight Entertainment, 2003).
Gamma Adjustment of Color Bands

green $\gamma = 0.5$

reduced green = incr. magenta
Gamma Adjustment of Color Bands

David Peters, producer, and representatives of the IA, The International Alliance of Theatrical Stage Employees, Moving Picture Technicians, Artists and Allied Crafts, on the set of Frozen Impact (PorchLight Entertainment, 2003).
Gamma Adjustment of Color Bands

blue $\gamma = 2$

Dave & IA image: blue band gamma = 2.0
Gamma Adjustment of Color Bands

David Peters, producer, and representatives of the IA, The International Alliance of Theatrical Stage Employees, Moving Picture Technicians, Artists and Allied Crafts, on the set of Frozen Impact (PorchLight Entertainment, 2003).
Gamma Adjustment of Color Bands

blue $\gamma=0.5$

reduced blue = incr. yellow

Dave & IA image: blue band gamma = 0.5

1 October 2018

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Gamma Adjustment of Color Bands

David Peters, producer, and representatives of the IA, The International Alliance of Theatrical Stage Employees, Moving Picture Technicians, Artists and Allied Crafts, on the set of Frozen Impact (PorchLight Entertainment, 2003).
RGB to LHS: A Perceptual Transformation

The eye has 3 types of photoreceptors: sensitive to red, green, or blue light.

The brain transforms RGB into separate brightness and color channels (e.g., LHS).
HSV Color Representation

Color vector \( p_0 \)

\( p_0 = (r_0, g_0, b_0) \)

Triangle:

lies in plane \( R+G+B=(r_0+g_0+b_0) \),

intersects \( R, G, \) & \( B \) axes at \( r_0+g_0+b_0 \), and contains \( p_0 \).

\[ \bar{v}_0 = \frac{1}{3}(r_0 + g_0 + b_0)(\hat{r} + \hat{g} + \hat{b}) \]

\[ \bar{s}_0 = \bar{p}_0 - \bar{v}_0 \]

\[ v_0 = \frac{\sqrt{3}}{3} \| \bar{v}_0 \| \]

\[ s_0 = \| \bar{s}_0 \| \]

\[ h_0 = \angle \left\{ \left[ (r_0 + g_0 + b_0) \hat{r} - \bar{v}_0 \right], \bar{s}_0 \right\} \]
A Fast RGB to HSV Algorithm

Given color $\mathbf{p} = [R \ G \ B]^T$ where $R, G, B \in \{0, \ldots, 255\}$, to compute $[h \ s \ v]^T$ where $s, v \in [0,1]$ and $h \in [0,360)$ the algorithm proceeds as follows:

1. Compute $[r \ g \ b] = [R \ G \ B]/255$.
2. Set $m = \min(r, g, b)$, $M = \max(r, g, b)$.
3. Set $v = M$.
4. Compute $C = M - m$.
5. If $C == 0$ then $s=0$, $h=0$. Return $[h \ s \ v]^T$.
6. $s = C/M$.
7. If $M==r$ then $h = ((g-b)/c) \mod 6$.
8. else if $M==g$ then $h = 2 + (b-r)/c$.
9. else $h = 4 + (r-g)/c$.
10. $h = 60h$.

Experiments with Matlab show this algorithm to be 3 times faster than Algorithm 1 and 1.13 faster than Algorithm 2. The numbers output by this one differ from the other two.

Reference: [HSL and HSV - Wikipedia, the free encyclopedia](https://en.wikipedia.org/wiki/HSL_and_HSV)
HSV to RGB Conversion

Therefore, the rotation matrix is

\[
A = \frac{\sqrt{6}}{6} \begin{bmatrix}
2 & 0 & \sqrt{2} \\
-1 & \sqrt{3} & \sqrt{2} \\
-1 & -\sqrt{3} & \sqrt{2}
\end{bmatrix}.
\]

Substitute that into the 2\textsuperscript{nd} equation on slide 94 to get:

\[
\begin{bmatrix}
s \end{bmatrix}_{\text{rgb}} = s \frac{\sqrt{6}}{6} \cos(h) \begin{bmatrix}
2 \\
-1
\end{bmatrix} + s \frac{\sqrt{3}}{2} \sin(h) \begin{bmatrix}
0 \\
1 \\
-1
\end{bmatrix} + 0 \frac{\sqrt{3}}{3} \begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix}
\]

\[
= s \frac{\sqrt{6}}{6} \cos(h) \begin{bmatrix}
2 \\
-1
\end{bmatrix} + s \frac{\sqrt{3}}{2} \sin(h) \begin{bmatrix}
0 \\
1 \\
-1
\end{bmatrix}.
\]

Finally, \([s]_{\text{rgb}}\) must be translated to the value vector to obtain the rgb color of \(p_0\):

\[
p_0 = [p]_{\text{rgb}} = [s]_{\text{rgb}} + [v]_{\text{rgb}},\text{ where } s_0 = [s]_{\text{rgb}} \text{ and } [v]_{\text{rgb}} = v_0 \text{ as def’d. on slide 81.}
\]
A Fast HSV to RGB Algorithm

Given vector $h^T = [h \ s \ v]$ where $h \in [0, 360)$, $s \in [0, 1]$, and $v \in [0, 1]$, to compute $p^T = [r \ g \ b]$ where $r, g, b \in \{0, \ldots, 255\}$:

1. $H = h/60$.
2. $C = v \cdot s$.
3. $D = v - C$.
4. $X = C \cdot (1 - |(H \mod 2) - 1|)$.
5. if $0 \leq H < 1$ then $[r \ g \ b] = [C \ X \ 0]$ else if $1 \leq H < 2$ then $[r \ g \ b] = [X \ C \ 0]$ else if $2 \leq H < 3$ then $[r \ g \ b] = [0 \ C \ X]$ else if $3 \leq H < 4$ then $[r \ g \ b] = [0 \ X \ C]$ else if $4 \leq H < 5$ then $[r \ g \ b] = [X \ 0 \ C]$ else if $5 \leq H < 6$ then $[r \ g \ b] = [C \ 0 \ X]$ else $[r \ g \ b] = [0 \ 0 \ 0]$.
6. $[r \ g \ b] = 255 \cdot [r+D \ g+D \ b+D]$

Reference: [HSL and HSV - Wikipedia, the free encyclopedia](https://en.wikipedia.org/wiki/HSL_and_HSV)
Saturation Adjustment

Original Dave & IA Image
Saturation Adjustment

saturation + 50%

All the colors become closer to pure primaries.
Saturation Adjustment
Saturation Adjustment

The r, g, & b histograms approach the value histogram as the color fades to grayscale.

saturation - 50%
Hue Shifting

[Image of a group of people standing in front of a yellow bus with a graph showing color histograms for R, Y, G, C, B, and M.]
Hue Shifting

The effects of a hue shift are nonlinear. They are difficult to characterize on the r, g, & b histograms.
Hue Shifting

hue + 120°
Hue Shifting

hue + 180°

R → C
Y → B
G → M
C → R
B → Y
M → G
Hue Shifting

hue + 240°
Hue Shifting

$\text{hue} + 300^\circ$

$\begin{align*}
R &\rightarrow M \\
Y &\rightarrow R \\
G &\rightarrow Y \\
C &\rightarrow G \\
B &\rightarrow C \\
M &\rightarrow B
\end{align*}$
Hue Shifting

hue + 360° = original

Original Dave & IA Image

R → R
Y → Y
G → G
C → C
B → B
M → M
Hue Shifting

The effect of a hue shift on the hue histogram is quite obvious...
Hue Shifting

... the entire histogram is shifting...

Hue + 60°
Hue Shifting

... and the shift is circular since the hue is a circular function – it is defined on a circle.
Hue Shifting

The part of the histogram that leaves one side appears on the other.
Hue Shifting

hue + 240°

R → B
Y → M
G → R
C → Y
B → G
M → C

Hue + 240°
Hue Shifting

![Image of people in colorful clothing]

hue + 300°

- R → M
- Y → R
- G → Y
- C → G
- B → C
- M → B

Graph showing Hue + 300° distribution.
Hue Shifting

hue + 360° = original

R → R
Y → Y
G → G
C → C
B → B
M → M

Hue + 0°
Linear Transformation of Color

EECE 4353 Image Processing
Vanderbilt University School of Engineering

1 October 2018 391999-2018 by Richard Alan Peters II
Color Correction via Linear Transformation

is a point process; the transformation is applied to each pixel as a function of its color alone.

\[ \mathbf{J}(r, c) = \Phi \left[ \mathbf{I}(r, c) \right], \quad \forall (r, c) \in \text{supp}(\mathbf{I}). \]

Each pixel is vector valued, therefore the transformation is a vector space operator.

\[
\mathbf{I}(r, c) = \begin{bmatrix} \mathbf{R}_I(r, c) \\ \mathbf{G}_I(r, c) \\ \mathbf{B}_I(r, c) \end{bmatrix}, \quad \mathbf{J}(r, c) = \begin{bmatrix} \mathbf{R}_J(r, c) \\ \mathbf{G}_J(r, c) \\ \mathbf{B}_J(r, c) \end{bmatrix} = \Phi \left\{ \mathbf{I}(r, c) \right\} = \Phi \left\{ \begin{bmatrix} \mathbf{R}_I(r, c) \\ \mathbf{G}_I(r, c) \\ \mathbf{B}_I(r, c) \end{bmatrix} \right\}.
\]
Color Vector Space Operators

Linear operators are matrix multiplications

\[
\begin{bmatrix}
  r_1 \\
  g_1 \\
  b_1
\end{bmatrix} =
\begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
  r_0 \\
  g_0 \\
  b_0
\end{bmatrix}
\]

Example of a nonlinear operator: gamma correction

\[
\begin{bmatrix}
  r_1 \\
  g_1 \\
  b_1
\end{bmatrix} = 255 \cdot
\begin{bmatrix}
  (r_0 / 255)^{1/\gamma_r} \\
  (g_0 / 255)^{1/\gamma_g} \\
  (b_0 / 255)^{1/\gamma_b}
\end{bmatrix}
\]
Linear Transformation of Color

\[
\begin{bmatrix}
  r_1 \\
  g_0 \\
  b_0
\end{bmatrix} =
\begin{bmatrix}
  r_1/r_0 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  r_0 \\
  g_0 \\
  b_0
\end{bmatrix}
\]

\[
\begin{bmatrix}
  175 \\
  75 \\
  175
\end{bmatrix} =
\begin{bmatrix}
  125 \\
  75 \\
  175
\end{bmatrix}
\]
Linear Transformation of Color

\[
\begin{bmatrix}
    r_0 \\
    g_1 \\
    b_0 \\
\end{bmatrix} =
\begin{bmatrix}
    1 & 0 & 0 \\
    0 & g_1/g_0 & 0 \\
    0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
    r_0 \\
    g_0 \\
    b_0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
    125 \\
    150 \\
    175 \\
\end{bmatrix} \rightarrow \begin{bmatrix}
    125 \\
    75 \\
    175 \\
\end{bmatrix}
\]
Linear Transformation of Color

\[
\begin{bmatrix}
  r_0 \\
  g_0 \\
  b_1 \\
\end{bmatrix} = \begin{bmatrix}
  1 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & b_1/b_0 \\
\end{bmatrix} \begin{bmatrix}
  r_0 \\
  g_0 \\
  b_0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
  125 \\
  75 \\
  225 \\
\end{bmatrix} = \begin{bmatrix}
  125 \\
  75 \\
  175 \\
\end{bmatrix}
\]
Linear Transformation of Color

\[
\begin{bmatrix}
    r_1 \\
g_1 \\b_1
\end{bmatrix} = \begin{bmatrix}
    r_1/r_0 & 0 & 0 \\
    0 & g_1/g_0 & 0 \\
    0 & 0 & b_1/b_0
\end{bmatrix}\begin{bmatrix}
r_0 \\
g_0 \\b_0
\end{bmatrix}
\]

\[
\begin{bmatrix}
    r_1 \\
g_1 \\b_1
\end{bmatrix} = \begin{bmatrix}
    175 \\
    150 \\
    225
\end{bmatrix}
\]

\[
\begin{bmatrix}
r_0 \\
g_0 \\b_0
\end{bmatrix} = \begin{bmatrix}
    125 \\
    75 \\
    175
\end{bmatrix}
\]
Color Transformation

Assume $J$ is a discolored version of image $I$ such that $J = \Phi[I]$. If $\Phi$ is linear then it is represented by a $3 \times 3$ matrix, $A$:

$$A = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}.$$

Then $J = AI$ or, more accurately, $J(r,c) = AI(r,c)$ for all pixel locations $(r,c)$ in image $I$. 
Color Transformation

If at pixel location \((r, c)\),

\[
\begin{bmatrix}
\rho_1 \\
\gamma_1 \\
\beta_1
\end{bmatrix}
\]

image \(I(r, c)\) and

\[
\begin{bmatrix}
\rho_J \\
\gamma_J \\
\beta_J
\end{bmatrix}
\]

image \(J(r, c)\),

then

\[
J(r, c) = AI(r, c), \text{ or }
\]

\[
\begin{bmatrix}
\rho_J \\
\gamma_J \\
\beta_J
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
\rho_1 \\
\gamma_1 \\
\beta_1
\end{bmatrix}
\]

\[
= \begin{bmatrix}
a_{11}\rho_1 + a_{12}\gamma_1 + a_{13}\beta_1 \\
a_{21}\rho_1 + a_{22}\gamma_1 + a_{23}\beta_1 \\
a_{31}\rho_1 + a_{32}\gamma_1 + a_{33}\beta_1
\end{bmatrix}.
\]
Color Transformation

The inverse transform $\Phi^{-1}$ (if it exists) maps the discolored image, $J$, back into the correctly colored version, $I$, i.e., $I = \Phi^{-1}[J]$. If $\Phi$ is linear then it is represented by the inverse of matrix $A$:

$$A^{-1} = \begin{bmatrix}
    a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + \\
    a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} \\
    a_{22}a_{33} - a_{23}a_{32} & a_{13}a_{32} - a_{12}a_{33} & a_{12}a_{23} - a_{13}a_{22} \\
    a_{23}a_{31} - a_{21}a_{33} & a_{11}a_{33} - a_{13}a_{31} & a_{13}a_{21} - a_{11}a_{23} \\
    a_{21}a_{32} - a_{22}a_{31} & a_{12}a_{31} - a_{11}a_{32} & a_{11}a_{22} - a_{12}a_{21}
\end{bmatrix}^{-1} \cdot \begin{bmatrix}
    a_{22}a_{33} - a_{23}a_{32} & a_{13}a_{32} - a_{12}a_{33} & a_{12}a_{23} - a_{13}a_{22} \\
    a_{23}a_{31} - a_{21}a_{33} & a_{11}a_{33} - a_{13}a_{31} & a_{13}a_{21} - a_{11}a_{23} \\
    a_{21}a_{32} - a_{22}a_{31} & a_{12}a_{31} - a_{11}a_{32} & a_{11}a_{22} - a_{12}a_{21}
\end{bmatrix}.$$
Color Correction

Assume we know $n$ colors in the discolored image, $J$, that correspond to another set of $n$ colors (that we also know) in the original image, $I$.

\[
\left\{ \begin{array}{c}
\rho_{J, k} \\
\gamma_{J, k} \\
\beta_{J, k}
\end{array} \right\}_{k=1}^{n} = \left\{ \begin{array}{c}
\rho_{I, k} \\
\gamma_{I, k} \\
\beta_{I, k}
\end{array} \right\}_{k=1}^{n}
\]

for $k = 1, \ldots, n$. 

known wrong colors

known correct colors

known correspondence
Color Correction

To remap the discolored image so that the result matches the original image in a linearly optimal way, we need to find the matrix, $A$, that minimizes

$$
\epsilon^2 = \sum_{k=1}^{n} \left\| \begin{bmatrix} \rho_{I, k} \\ \gamma_{I, k} \\ \beta_{I, k} \end{bmatrix} - A^{-1} \begin{bmatrix} \rho_{J, k} \\ \gamma_{J, k} \\ \beta_{J, k} \end{bmatrix} \right\|^2
$$
Color Correction

To find the solution of this problem, let

\[
Y = \begin{bmatrix}
\rho_{1,1} & \cdots & \rho_{1,n} \\
\gamma_{1,1} & \cdots & \gamma_{1,n} \\
\beta_{1,1} & \cdots & \beta_{1,n}
\end{bmatrix}, \quad \text{and} \quad X = \begin{bmatrix}
\rho_{J,1} & \cdots & \rho_{J,n} \\
\gamma_{J,1} & \cdots & \gamma_{J,n} \\
\beta_{J,1} & \cdots & \beta_{J,n}
\end{bmatrix}.
\]

Then \(X\) and \(Y\) are known \(3 \times n\) matrices such that

\[
Y \approx A^{-1} X,
\]

where \(A\) is the \(3 \times 3\) matrix that we want to find.
Color Correction

The linearly optimal solution is the least mean squared solution that is given by

\[ B = A^{-1} = YX^T \left( XX^T \right)^{-1} \]

where \( X^T \) represents the transpose of matrix \( X \).

Notes:  
1. \( n \), the number of color pairs, must be \( \geq 3 \),
2. \( XX^T \) must be invertible, \( i.e., \) \( \text{rank}(XX^T) = 3 \),
3. If \( n=3 \), then \( X^T(XX^T)^{-1} = X^{-1} \).  

important
Color Correction

The linearly optimal solution that is given by

\[ B = A^{-1} = YX^T \left( XX^T \right)^{-1} \]

where \( X^T \) represents the transpose of matrix \( X \).

Notes:
1. \( n \), the number of color pairs, must be \( \geq 3 \),
2. \( XX^T \) must be invertible, \( i.e., \text{rank}(XX^T) = 3 \),
3. If \( n=3 \), then \( X^T(XX^T)^{-1} = X^{-1} \).
Color Correction

The linearly optimal solution is a least mean squared solution that is given by

\[ B = A^{-1} = YX^T \left( XX^T \right)^{-1} \]

where \( X^T \) represents the transpose of matrix \( X \).

Notes:
1. \( n \), the number of color pairs, must be \( \geq 3 \),
2. \( XX^T \) must be invertible, \( i.e., \) \( \text{rank}(XX^T) = 3 \),
3. If \( n=3 \), then \( X^T(XX^T)^{-1} = X^{-1} \).
Color Correction

Then the image is color corrected by performing

\[ I(r,c) = B \cdot J(r,c), \text{ for all } (r,c) \in \text{supp}(J). \]

In **Matlab** this is easily performed by

```matlab
>> I = reshape(((B*(reshape(double(J),R*C,3))')'),R,C,3);
>> m = min(I(:));
>> M = max(I(:));
>> I = uint8(255*(I-m)/(M-m));
```

where \( B=A^{-1} \) is computed directly through the LMS formula on the previous page, and \( R \) & \( C \) are the number of rows and columns in the image.
Linear Color Correction

NASA Summer Faculty Fellows at Ellington Air Force Base, Houston, TX, July 2002. Airplane is a T-38.

Original Image  

“Aged” Image
Color Mapping 1

Original Image

“Aged” Image
Color Mapping 2

Original Image

“Aged” Image
Color Mapping 3

Original Image

“Aged” Image
Color Mapping 4

Original Image

“Aged” Image
The aging process was a transformation, $\Phi$, that mapped:

\[
\begin{bmatrix}
17 \\ 122 \\ 114
\end{bmatrix} = \Phi \begin{bmatrix}
17 \\ 121 \\ 171
\end{bmatrix} = \begin{bmatrix}
222 \\ 222 \\ 218
\end{bmatrix} = \Phi \begin{bmatrix}
222 \\ 222 \\ 218
\end{bmatrix} = \begin{bmatrix}
240 \\ 185 \\ 218
\end{bmatrix} = \Phi \begin{bmatrix}
240 \\ 171 \\ 160
\end{bmatrix} = \begin{bmatrix}
236 \\ 227 \\ 166
\end{bmatrix}
\]
Color Transformations

To undo the process we need to find, $\Phi^{-1}$, that maps:

\[
\begin{pmatrix}
17 \\
121 \\
171
\end{pmatrix} = \Phi^{-1}
\begin{pmatrix}
17 \\
122 \\
114
\end{pmatrix},
\begin{pmatrix}
222 \\
222 \\
218
\end{pmatrix} = \Phi^{-1}
\begin{pmatrix}
222 \\
222 \\
185
\end{pmatrix},
\begin{pmatrix}
240 \\
230 \\
166
\end{pmatrix} = \Phi^{-1}
\begin{pmatrix}
240 \\
171 \\
160
\end{pmatrix},
\begin{pmatrix}
240 \\
171 \\
103
\end{pmatrix} = \Phi^{-1}
\begin{pmatrix}
240 \\
171 \\
106
\end{pmatrix}.\]
Correction Using 3 Mappings

\[ B = A^{-1} = YX^{-1} \]

\[ X = \begin{bmatrix} 222 & 17 & 240 \\ 222 & 122 & 171 \\ 185 & 114 & 103 \end{bmatrix} \]

\[ Y = \begin{bmatrix} 222 & 17 & 240 \\ 222 & 121 & 171 \\ 218 & 171 & 160 \end{bmatrix} \]
Correction Using 3 Mappings

\[ B = A^{-1} = YX^{-1} \]

original

\[ \mathbf{X} = \begin{bmatrix} 222 & 17 & 240 \\ 222 & 122 & 171 \\ 185 & 114 & 103 \end{bmatrix} \]

corrected

\[ \mathbf{Y} = \begin{bmatrix} 222 & 17 & 240 \\ 222 & 121 & 171 \\ 218 & 171 & 160 \end{bmatrix} \]
Another Correction Using 3 Mappings

\[ B = A^{-1} = YX^{-1} \]

\[
X = \begin{bmatrix}
222 & 17 & 236 \\
222 & 122 & 227 \\
185 & 114 & 106
\end{bmatrix}
\]

\[
Y = \begin{bmatrix}
222 & 17 & 240 \\
222 & 121 & 230 \\
218 & 171 & 166
\end{bmatrix}
\]
Another Correction Using 3 Mappings

\[ B = A^{-1} = YX^{-1} \]
Correction Using All 4 Mappings

\[ B = A^{-1} = YX^T(XX^T)^{-1} \]

\[
X = \begin{bmatrix}
222 & 17 & 236 & 240 \\
222 & 122 & 227 & 171 \\
185 & 114 & 106 & 103 \\
\end{bmatrix}
\]

\[
Y = \begin{bmatrix}
222 & 17 & 240 & 240 \\
222 & 121 & 230 & 171 \\
218 & 171 & 166 & 160 \\
\end{bmatrix}
\]
Correction Using All 4 Mappings

\[ B = A^{-1} = YX^T (XX^T)^{-1} \]

\[ X = \begin{bmatrix} 222 & 17 & 236 & 240 \\ 222 & 122 & 227 & 171 \\ 185 & 114 & 106 & 103 \end{bmatrix} \]

\[ Y = \begin{bmatrix} 222 & 17 & 240 & 240 \\ 222 & 121 & 230 & 171 \\ 218 & 171 & 166 & 160 \end{bmatrix} \]
Random Sampling of Color Values

\[
\begin{align*}
>> \, rr &= \text{round}(R*\text{rand}([1 \ n])); \\
>> \, rc &= \text{round}(C*\text{rand}([1 \ n])); \\
>> \, \text{idx} &= [rr;rc]; \\
>> \, Y(:,1) &= \text{diag}(I(rr,rc,1)); \\
>> \, Y(:,2) &= \text{diag}(I(rr,rc,2)); \\
>> \, Y(:,3) &= \text{diag}(I(rr,rc,3)); \\
>> \, X(:,1) &= \text{diag}(J(rr,rc,1)); \\
>> \, X(:,2) &= \text{diag}(J(rr,rc,2)); \\
>> \, X(:,3) &= \text{diag}(J(rr,rc,3));
\end{align*}
\]

- \( R \) = number of rows in image
- \( C \) = number of columns in image
- \( n \) = number of pixels to select

\text{rand([1 \ n]): 1 \times n matrix of random numbers between 0 and 1.}

\text{diag(I(rr,rc,1)): vector from main diagonal of matrix I(rr,rc,1).}
Correction Using 128 Mappings

\[ B = A^{-1} = YX^T(XX^T)^{-1} \]

\[
\begin{bmatrix}
111 & 235 \\
103 & 233 \\
22 & 210
\end{bmatrix}
\]

\[
\begin{bmatrix}
111 & 234 \\
102 & 233 \\
71 & 229
\end{bmatrix}
\]
Correction Using 128 Mappings

\[
B = A^{-1} = YX^T \left(XX^T\right)^{-1}
\]

\[
X = \begin{bmatrix}
111 & 235 \\
103 & \cdots & 233 \\
22 & 210
\end{bmatrix}
\]

\[
Y = \begin{bmatrix}
111 & 234 \\
102 & \cdots & 233 \\
71 & 229
\end{bmatrix}
\]
Correction Using 128 Mappings

\[ B = A^{-1} = YX^T (XX^T)^{-1} \]

original

\[
X = \begin{bmatrix}
111 \\
103 \\
22
\end{bmatrix}
\quad Y = \begin{bmatrix}
111 \\
102 \\
71
\end{bmatrix}
\]

corrected
Correction Using 4 Mappings

\[ B = A^{-1} = YX^T \left( XX^T \right)^{-1} \]

original

corrected

for comparison:

\[
\begin{bmatrix}
222 & 17 & 236 & 240 \\
222 & 122 & 227 & 171 \\
185 & 114 & 106 & 103
\end{bmatrix}
\]

\[
\begin{bmatrix}
222 & 17 & 240 & 240 \\
222 & 121 & 230 & 171 \\
218 & 171 & 166 & 160
\end{bmatrix}
\]
Linear Color Transformation Program

```matlab
function J = LinTrans(I,A)

[R C B] = size(I);
I = double(I);
J = reshape(((A*(reshape(I,R*C,3))')'),R,C,3);
return;
```

This function returns an image of class double. To get a good uint8 you may have to linearly scale the result as shown on slide 53.